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A STUDY ON THE PROPAGATION OF THE TSUNAMI WAVE

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ABSTRACT

Water waves have broad appeal as a scientific topic.In this paper "water waves" refers to the waves that occur on the free surface of a body of water like an ocean under the force of gravity. This paper is based on the process of tsunami evolution during its generation under the effect of the variable velocities of submarine landslides based on a two dimensional slide model. By using Laplace transform in time and Fourier transform in space, tsunami wave forms within the linearized shallow water theory for constant water depth. Effects of water depths on the amplification factor of the tsunami generation by submarine slump and different propagation lengths,widths has been studied in this paper.

Keywords- *Mathematical modelling of tsunami, water wave, tsunami amplitude, water wave theory, Laplace and Fourier Transforms.*

I. INTRODUCTION

The word tsunami originates from a combination of two Japanese words 'tsu' and 'nami' translated as a 'wave in the harbour'. A tsunami is a very long wavelength wave, generated by an underwater earthquake or landslide. A tsunami is a train of high waves which includes tides, storm and tropical cyclones. Almost all large earthquakes are the major reasons behind a tsunami.

Indian Ocean had been also stroked by this unimaginable disaster in the morning of December 26, 2004.Wiping out more than 275,000 human life at a stroke from the face of the earth. It was the killer Tsunami, that originated its journey at the epicentre of the earthquake (of intensity 9.2) near Banda Aceh in Indonesia and travelled as long as to Port Elizabeth in South Africa, covering a distance of more than 8,000 km and bringing unprecedented devastation to the countries like Indonesia, Thailand, Sri Lanka, India and others.

The whole world was shocked saddened and felt helpless. There were efforts to study different aspects of the Tsunami and other oceanic waves with special emphasis on the nonlinear connection of this problem by the scientists. Till the very end of the twentieth century tsunami waves (or 'waves in a harbour', translated from Japanese) were considered an extremely rare and exotic natural phenomenon, originating in the ocean and unexpectedly falling upon the seaside.

Waves that regularly devastate the coasts of oceanic islands and are called tsunami in Japan have been known for several centuries. The European civilization first encountered such catastrophic waves in 1755, when an exceptionally strong earthquake took place in the Atlantic Ocean near the coast of Portugal and gave rise to a tsunami wave that immediately killed over 50,000 people in the blooming city of Lisbon, which was about a quarter of the city's population. During the past 10 years (not counting the tragedy caused by the Indonesian tsunami in 2004) tsunami waves in the Pacific Ocean took the lives of more than 10,000 people.

According to UNESCO information, by the year 2010 residents of the coasts of oceans and seas will represent about 70% of the total population of our planet. One should add persons visiting numerous seaside resorts, those who like to celebrate the New Year on exotic oceanic islands and, also, individuals seeking maritime adventures. All these people may happen to be within the reach of one of the oceanic catastrophes, of which tsunami waves are the most dangerous. Before explaining the generation of Tsunami we have to understand the composition of earth and causes of earthquakes.

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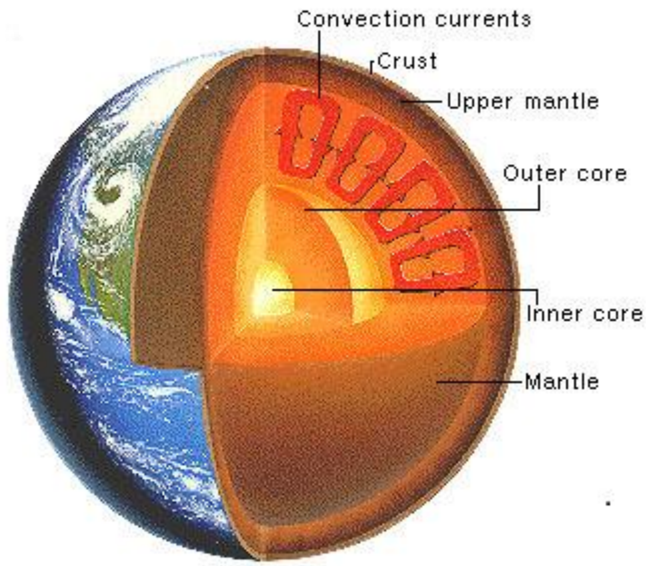
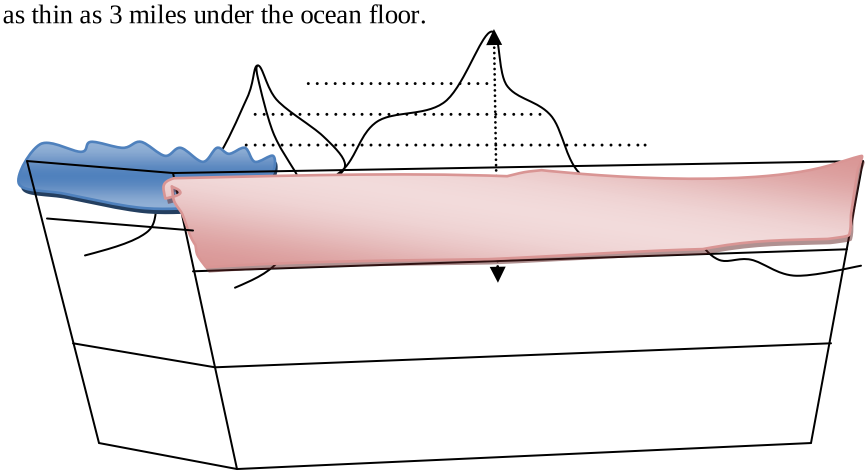
II. DISCUSSION

The Earth and its Composition



Let us consider that the earth is like an apple and consists of a skin called "meaty" part and a core or pit. The core of the earth is a solid metal made by iron or nickel surrounded by hot liquid metals. The mantle of the earth is a hot part somewhat soupy mass of metalled rock called 'magma'. Crust is the skin of the planet. It is very hard surface of the earth on which we live. The crust is not equally thick all around the earth. It is as deep as 25 miles under the surface of the continents and as thin as 3 miles under the ocean floor.



Until recently the crust was assumed to be solid rock, but we can see that it is cracked into separate sections called tectonic plates. Some of them are so large that they determine the boundaries of entire continents or oceans one of the plates supports all of the United States, and the whole Pacific Ocean sits on another. Other sections are smaller, supporting only part of a continent or a small group of islands, like the plate under the Caribbean.

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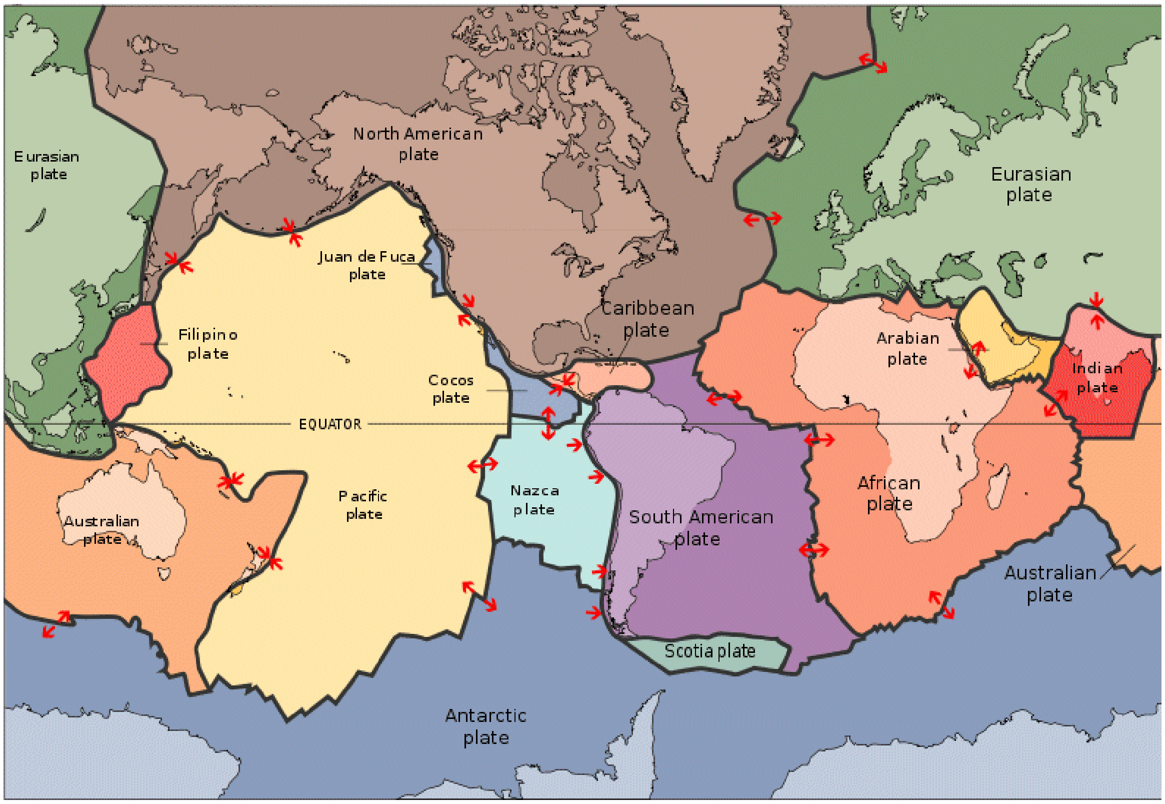
The main Tectonic plates and boundaries:



The main source of Tsunami:

Almost all large earthquakes occur at the boundary of tectonic plates, where one plate is sliding over, under, or past another. Seismologists classify earthquakes into three types of faults, depending on how the relative motion of adjacent plates affects the shape of the solid earth.

- In a thrust fault, one tectonic plate moves up and over an adjacent plate.
- In a normal fault, one plate moves down relative to an adjacent plate.
- In a strike-slip fault, two plates slide past each other horizontally, with neither plate being raised or lowered significantly.
- Reverse normal faults and thrust faults are closely related.

The separate tectonic plates flowing over the magma don't stay put but move around at a snail's pace, at only 50 millimetres (2 inches) a year. As they move toward each other, one plate may hit another (top left), slide along it (top right), or even duck under it in a movement called sub duct ion (bottom).

Tsunami generation is mainly caused by sharp vertical displacements of landslide (slumps). It is related to volcanic eruption also. Recently, particular interest has been shown in the possibility of catastrophic tsunami waves arising due to bodies falling into the ocean from outer space. Such waves are conventionally termed Cosmo genic tsunamis in modern scientific literature.

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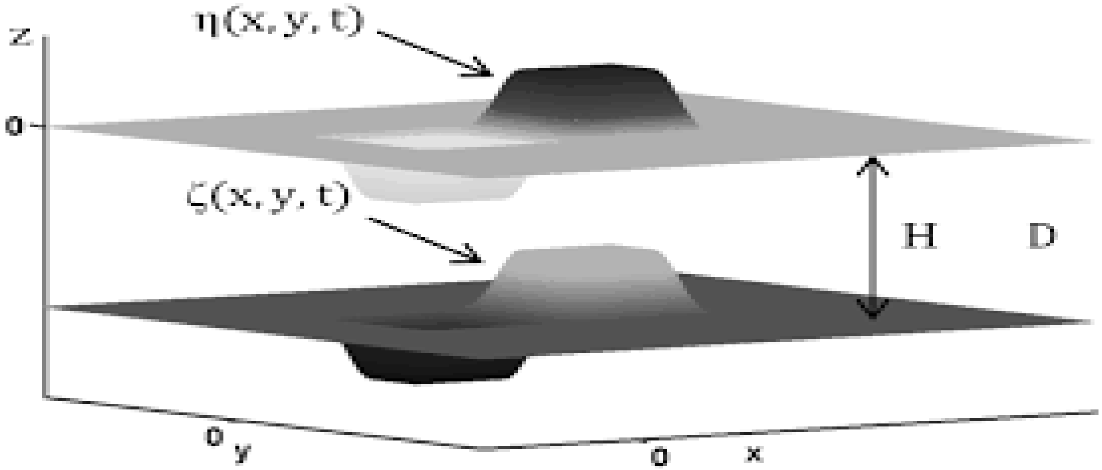
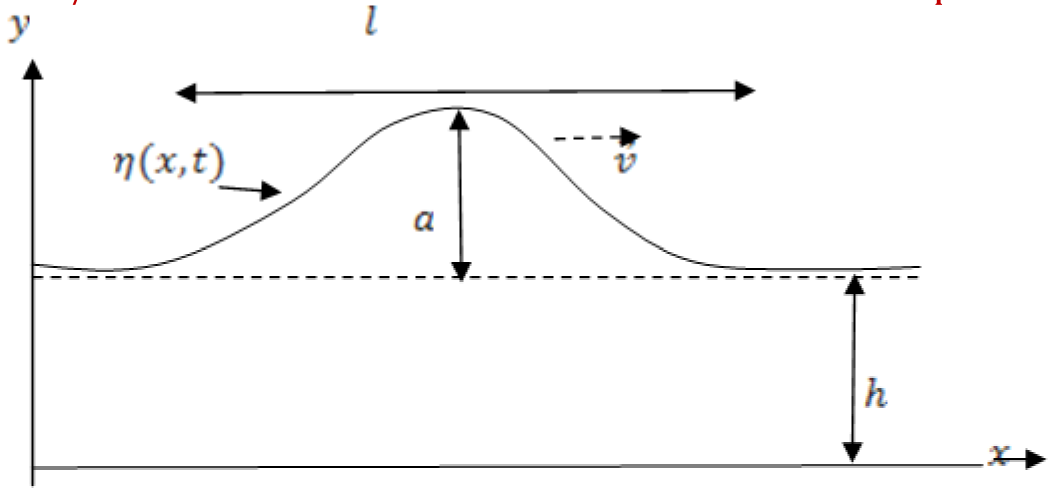
The Mathematical modelling of Tsunami waves:



Consider the one-dimensional (x-direction) wave motion of an incompressible and in viscid fluid (water) in a shallow channel of height h, and of sufficient width with uniform cross-section leading to the formation of a solitary wave propagating under gravity. Let the length of the wave be land the maximum value of its amplitude$\eta(x,t)$, above the horizontal surface be a (see Fig.).



Then we can introduce two natural small parameters into the problem ε and δ as

$$\varepsilon = \frac{a}{h} \approx 10^{-4} << 1 \text{ And } \delta = \frac{h}{l} \approx 10^{-2} << 1$$

Then we can proceed with the analysis as follows:

Equation of Motion:

The fluid motion can be described by the velocity vector

$$\boldsymbol{V}(x,y,t) = u(x,y,t)\boldsymbol{i} + v(x,y,t)\boldsymbol{j} \qquad (1)$$

Where $\boldsymbol{i}$ and $\boldsymbol{j}$ are the unit vectors along the horizontal and vertical directions, respectively. As the motion is irrotational, we have

$$\boldsymbol{\nabla} \times \boldsymbol{V} = 0 \qquad (2)$$

Consequently, we can introduce the velocity potential $\varphi(x,y,t)$ by the relation

$$\boldsymbol{V} = \boldsymbol{\nabla}\varphi \qquad (3)$$

(i) Conservation of Density

The system obviously admits the following conservation law for the mass density $\rho(x,y,t)$ of the fluid,

$$\frac{d\rho}{dt} = \rho t + \nabla(\rho v) = 0 \qquad (4)$$

Where $\boldsymbol{V}(x,y,t)$ is the velocity vector of the fluid. As ρ is a constant, from (4) we have

$$\boldsymbol{\nabla} \cdot \boldsymbol{V} = 0 \qquad (5)$$

Then using (3) in (5), we find that φ obeys the Laplace equation

$$\nabla^2\varphi(x,y,t) = 0 \qquad (6)$$

(ii) Euler's Equation:

As the density of the fluid $\rho = \rho_0$ = constant, using Newton's law for the rate of change of momentum, we can write

$$\frac{d\vec{V}}{dt} = \frac{\partial\vec{V}}{\partial t} + \left(\vec{V} \bullet \nabla\right)\vec{V} = -\frac{1}{\rho_0}\nabla p - gj \qquad (7)$$

Where $p = p(x, y, t)$ is the pressure at the point (x, y) and g is the acceleration due to gravity, which is acting vertically downwards (here $\boldsymbol{j}$ is the unit vector along the vertical direction). Making use of (3) in (7), we obtain (after one integration)

$$\frac{\partial \phi}{\partial t} + \frac{1}{2}(\vec{\nabla}\phi)^2 + \frac{p}{\rho_0} + gy = 0 \qquad (8)$$

(iii) Boundary Conditions

The above two equations (6) and (8) for the velocity potential $\varphi(x, y, t)$ of the fluid have to be supplemented by appropriate boundary conditions, by taking into account the fact that

(a) The horizontal bed at $y = 0$ is hard and

(b) The upper boundary $y = y(x, t)$ is a free surface.

As a result

(a) The vertical velocity at $y = 0$ vanishes, $v(x, 0, t) = 0$ (9)

Which implies (using (1) and (3)) $\varphi_y(x, 0, t) = 0$ (10)

(b) As the upper boundary is free, let us specify it by $y = h + \eta(x, t)$

Then at the point $x = x_1$ *and* $y = y_1 \equiv y(x, t)$, we can write

$$\frac{dy_1}{dt} = \left[\frac{\partial}{\partial t} + u_1\frac{\partial}{\partial x}\right](h + \eta(x,t)) = \frac{\partial \eta}{\partial t} + u_1\frac{\partial \eta}{\partial x} = v_1. \qquad (11)$$

Since $v_1 = \varphi_{1y}, u_1 = \varphi_{1x}$, the last two parts of (11) can be rewritten as

$$\varphi_{1y} = \eta t + \eta_x \varphi_{1x} \qquad (12)$$

(c) Similarly at $y = y_1$ the pressure $p_1 = 0$. Then from (8), it follows that

$$u_{1t} + u_1 u_{1x} + v_1 v_{1x} + g\eta_x = 0 \qquad (13)$$

Thus the motion of the surface of water wave is essentially specified by the Laplace equation (6) and (8) along with one fixed boundary condition (10) and two variable nonlinear boundary conditions (12) and (13).

One has to then solve the Laplace equation subject to these boundary conditions [25].

There are many approximations which can be considered for the full water-wave equations. One is the system of Boussinesq equations that retains nonlinearity and dispersion up to a certain order. This model is used to study transient varying bottom problems. Fuhrman and Madsen [26] and Zhao et al. [33] presented a developed numerical model based on the highly accurate values. Boussinesq-type formulation is subject to exact expressions for the kinematic and dynamic free surface conditions. Their results show that the model was capable of treating the full life cycle of tsunami evolution, from the initial generation of bottom movements, to the subsequent propagation and through the final run-up process. Reasonable computational efficiency has been demonstrated in their work, which made the model attractive for practical coastal engineering studies. Another one is the system of nonlinear shallow-water equations that retains nonlinearity but no dispersion. Solving this problem is a difficult task due to the nonlinearities and the priori unknown free surface. The simplest one is the system of linear shallow-water equations. The concept of shallow water is based on the smallness of the ratio between water depth and wave length. In the case of tsunamis propagating on the surface of deep oceans, one can consider that shallow-water theory is appropriate because the water depth (typically several kilometres) is much smaller than the wave length (typically several hundred kilometres), which is reasonable and usually true for most tsunamis triggered by submarine earthquakes, slumps and slides, [27, 28].

Mathematical modelling of waves generated by vertical and lateral displacements of ocean bottom using the combined Fourier–Laplace transforms of the Laplace equation analytically is the simplest way of studying tsunami

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development. All our studies took into account constant depth for which the Laplace and Fast Fourier Transform (FFT) methods can be applied.

III. CONCLUSION

In this paper we presented a review of the main physical dynamics of Tsunami generation which is caused by landslides and tectonic plates. The tsunami wave form has two large peaks of amplitudes, one is in front of the block due to forward sliding and the other one is behind the block due to spreading of the depletion zone. We studied the effect of variable velocities of submarine block slide on the tsunami generation. It is also explained that the leading tsunami amplitudes are reduced in both the cases due to the geometric spreading and the dispersion.

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